JEE Advanced

Single Correct Answer Type

1. The half-life of radioactive radon is 3.8 days. The time at the end of which 1/20th of the radon sample will remain undecayed is (given $\log_{10} e = 0.4343$)
   a. 3.8 days **b.** 16.5 days **c.** 33 days **d.** 76 days
   (IIT-JEE 1981)

2. Beta rays emitted by a radioactive material are
   a. electromagnetic radiations
   b. the electrons orbiting around the nucleus
   c. charged particles emitted by the nucleus
   d. neutral particles **(IIT-JEE 1983)**

3. The equation $4{}_1^1\text{H} \rightarrow {}_2^4\text{He}^2 + 2e^- + 26\text{ MeV}$ represents
   a. β-decay **b.** γ-decay **c.** fusion **d.** fission
   (IIT-JEE 1983)

4. During a negative beta decay,
   a. an atomic electron is ejected
   b. an electron which is already present within the nucleus is ejected
   c. a neutron in the nucleus decays emitting an electron
   d. a part of the binding energy of the nucleus is converted into an electron **(IIT-JEE 1987)**

5. During a nuclear fusion reaction,
   a. a heavy nucleus breaks into two fragments by itself
   b. a light nucleus bombarded by thermal neutrons breaks up
   c. a heavy nucleus bombarded by thermal neutrons breaks up
   d. two light nuclei combine to give a heavier nucleus and possibly other products **(IIT-JEE 1987)**

6. Four physical quantities are listed in Column I. Their values are listed in Column II in a random order.

Column I	Column II
p. Thermal energy of air molecules at room temperature	**i.** 0.02 eV
q. Binding energy of heavy nuclei per nucleon	**ii.** 2 eV
r. X-ray photon energy	**iii.** 10 keV
s. Photon energy of visible light	**iv.** 7 MeV

The correct matching of Column I and Column II is given by

a. p → i, q → iv, r → iii, s → ii
b. p → i, q → iii, r → ii, s → iv
c. p → ii, q → i, r → iii, s → iv
d. p → ii, q → iv, r → i, s → iii **(IIT-JEE 1987)**

7. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

a. 6 h b. 12 h c. 24 h d. 28 h

(IIT-JEE 1988)

8. The decay constant of a radioactive sample is λ. The half-life and mean-life of the sample are, respectively, given by

a. $1/\lambda$ and $(\ln 2)/\lambda$ b. $(\ln 2)\lambda$ and $1/\lambda$
c. $\lambda(\ln 2)$ and $1/\lambda$ d. $\lambda/(\ln 2)$ and $1/\lambda$

(IIT-JEE 1989)

9. A star initially has 10^{40} deuterons. It produces energy via the processes ${}_1H^2 + {}_1H^2 \rightarrow {}_1H^2 + p$ and ${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of

a. 10^6 s b. 10^8 s c. 10^{12} s d. 10^{16} s

[The mass of the nuclei are as follows:
$M(H^2) = 2.014$ amu, $M(n) = 1.008$ amu,
$M(p) = 1.007$ amu, $M(He^4) = 4.001$ amu]

(IIT-JEE 1993)

10. Fast neutrons can easily be slowed down by

a. the use of lead shielding
b. passing them through water
c. elastic collision with heavy nuclei
d. applying a strong electric field **(IIT-JEE 1994)**

11. Consider α-particles, β-particles, and γ-rays, each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are:

a. α, β, γ b. α, γ, β c. β, γ, α d. γ, β, α

(IIT-JEE 1994)

12. Masses of two isobars ${}_{29}Cu^{64}$ and ${}_{30}Zn^{64}$ are 63.9298 u and 63.9292 u, respectively. It can be concluded from these data that

a. both the isobars are stable
b. Zn^{64} is radioactive, decaying to Cu^{64} through β-decay
c. Cu^{64} is radioactive, decaying to Zn^{64} through γ-decay
d. Cu^{64} is radioactive, decaying to Zn^{64} through β-decay

(IIT-JEE 1997)

13. The half-life of ${}^{131}I$ is 8 days. Given a sample of ${}^{131}I$ at time $t = 0$, we can assert that

a. no nucleus will decay before $t = 4$ days
b. no nucleus will decay before $t = 8$ days
c. all nuclei will decay before $t = 16$ days
d. a given nucleus may decay at any time after $t = 0$

(IIT-JEE 1998)

14. In hydrogen spectrum, the wavelength of Hα line is 656 nm, whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is

a. 2×10^8 m s^{-1} b. 2×10^7 m s^{-1}
c. 2×10^6 m s^{-1} d. 2×10^5 m s^{-1}

(IIT-JEE 1999)

15. Order of magnitude of density of uranium nucleus is [$m_p = 1.67 \times 10^{-27}$ kg]

a. 10^{20} kg m^{-3} b. 10^{17} kg m^{-3}
c. 10^{14} kg m^{-3} d. 10^{11} kg m^{-3}

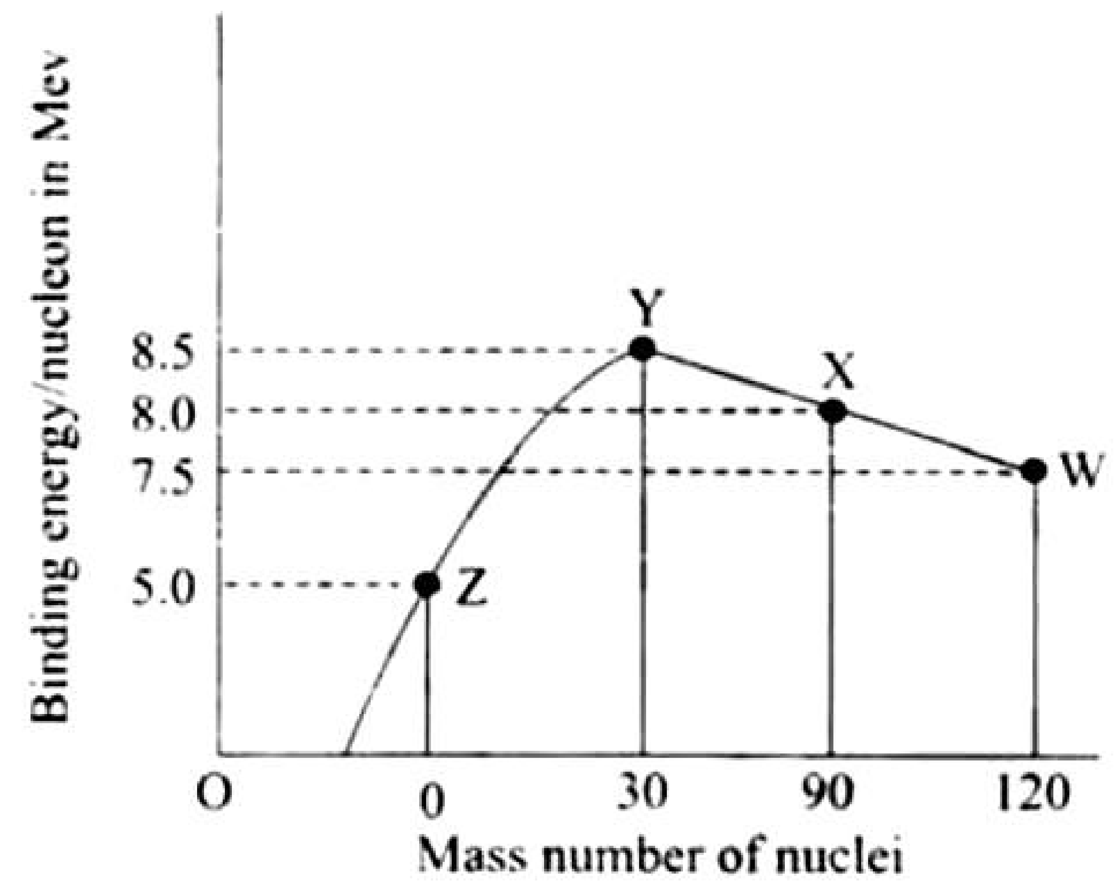
(IIT-JEE 1999)

16. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is

a. nitrogen b. carbon c. boron d. oxygen

(IIT-JEE 1999)

17. Binding energy per nucleon vs. mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is



a. $Y \rightarrow 2Z$ b. $W \rightarrow X + Z$
c. $W \rightarrow 2Y$ d. $X \rightarrow Y + Z$

(IIT-JEE 1999)

18. The half-life period of a radioactive element X is same as the mean lifetime of another radioactive element Y. Initially, both of them have the same number of atoms. Then,

a. X and Y have the same decay rate initially
b. X and Y decay at the same rate always
c. Y will decay at a faster rate than X
d. X will decay at a faster rate than Y

(IIT-JEE 1999)

19. Which of the following is a correct statement?

a. Beta rays are same as cathode rays.
b. Gamma rays are high-energy neutrons.
c. Alpha particles are singly ionized helium atoms.
d. Protons and neutrons have exactly the same mass.

(IIT-JEE 1999)

20. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time

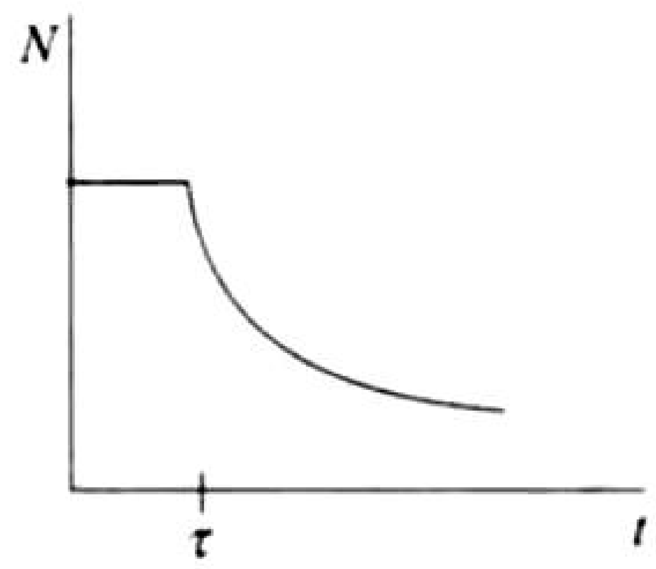
a. $\frac{1}{10\lambda}$ b. $\frac{1}{11\lambda}$ c. $\frac{11}{10\lambda}$ d. $\frac{1}{9\lambda}$

(IIT-JEE 2000)

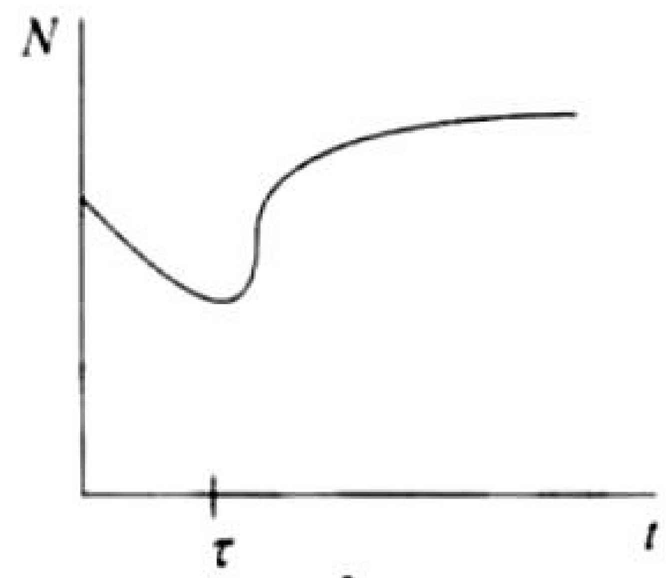
21. The electron emitted in beta radiation originates from

a. inner orbits of atoms
b. free electrons existing in nuclei
c. decay of a neutron in a nucleus
d. photon escaping from the nucleus **(IIT-JEE 2001)**

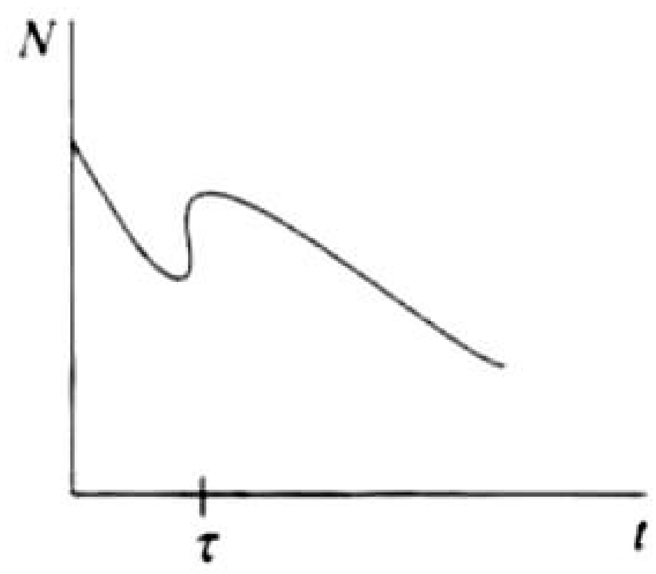
22. A radioactive sample consists of two distinct species having equal number of atoms initially. The mean life-time of one species is τ and that of the other is 5τ. The decay products in both cases are stable. A plot is made of the total number of radioactive nuclei as a function of time. Which of the following figures best represents the form of this plot?



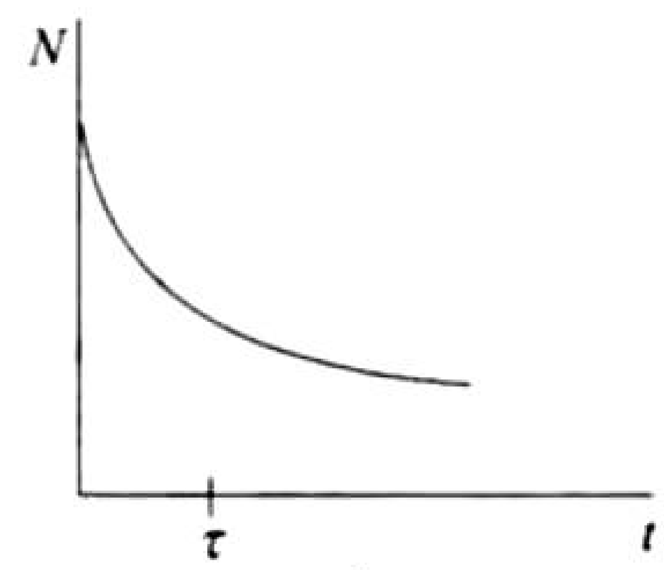
a.



b.



c.



d.

(IIT-JEE 2001)

23. The half-life of ^{215}At is 100 μs. The time taken for the radioactivity of a sample of ^{215}At to decay to 1/16th of its initial value is

a. 400 μs **b.** 6.3 μs **c.** 40 μs **d.** 300 μs

(IIT-JEE 2002)

24. Which of the following processes represents a γ-decay?

a. $^{A}X_{Z} + \gamma \rightarrow {}^{A}X_{Z-1} + a + b$

b. $^{A}X_{Z} + {}^{1}n_{0} \rightarrow {}^{A-3}X_{Z-2} + c$

c. $^{A}X_{Z} \rightarrow {}^{A}X_{Z} + f$

d. $^{A}X_{Z} + e_{-1} \rightarrow {}^{A}X_{Z-1} + g$ **(IIT-JEE 2002)**

25. For uranium nucleus, how does its mass vary with volume?

a. $m \propto V$ **b.** $m \propto 1/V$

c. $m \propto \sqrt{V}$ **d.** $m \propto V^2$

(IIT-JEE 2003)

26. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

a. 4.4 MeV **b.** 5.4 MeV

c. 5.6 MeV **d.** 6.5 MeV

(IIT-JEE 2003)

27. A 280-day-old radioactive substance shows an activity of 6000 dps. 140 days later its activity becomes 3000 dps. What was its initial activity?

a. 20000 dps **b.** 24000 dps

c. 120000 dps **d.** 6000 dps

(IIT-JEE 2004)

28. If a star can convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nuclei is [Mass of He nucleus is 4.0026 amu and mass of oxygen nucleus is 15.9994 amu]

a. 7.6 MeV **b.** 56.12 MeV

c. 10.24 MeV **d.** 23.9 MeV

(IIT-JEE 2005)

29. $^{221}_{87}$Ra is a radioactive substance having half-life of 4 days. Find the probability that a nucleus undergoes decay after two half-lives.

a. 1 **b.** $\frac{1}{2}$ **c.** $\frac{3}{4}$ **d.** $\frac{1}{4}$

(IIT-JEE 2006)

30. In the options given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is

a. $E({}^{236}_{92}\text{U}) > E({}^{137}_{53}\text{I}) + E({}^{97}_{39}\text{Y}) + 2E(\text{n})$

b. $E({}^{236}_{92}\text{U}) < E({}^{137}_{53}\text{I}) + E({}^{97}_{39}\text{Y}) + 2E(\text{n})$

c. $E({}^{236}_{92}\text{U}) < E({}^{140}_{56}\text{Ba}) + E({}^{94}_{36}\text{Kr}) + 2E(\text{n})$

d. $E({}^{236}_{92}\text{U}) = E({}^{140}_{56}\text{Ba}) + E({}^{94}_{36}\text{Kr}) + 2E(\text{n})$

(IIT-JEE 2007)

31. A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be

a. 20 years and 5 years, respectively.

b. 20 years and 10 years, respectively.

c. 10 years each

d. 5 years each **(IIT-JEE 2008)**

Multiple Correct Answers Type

1. From the following equations, pick out the possible nuclear fusion reaction:

a. ${}_{6}C^{13} + {}_{1}H^{1} \rightarrow {}_{6}C^{14} + 4.3$ MeV

b. ${}_{6}C^{12} + {}_{1}H^{1} \rightarrow {}_{7}C^{13} + 2$ MeV

c. ${}_{7}C^{14} + {}_{1}H^{1} \rightarrow {}_{8}O^{15} + 7.3$ MeV

d. ${}_{92}U^{235} + {}_{0}n^{1}$
$\rightarrow {}_{54}Xe^{104} + {}_{38}Sr^{94} + {}_{0}n^{1} + {}_{0}n^{1} + y + 200$ MeV

(IIT-JEE 1984)

2. Which of the following statement(s) is(are) correct?

a. The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons.

b. The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.

c. In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 a.m.u.).

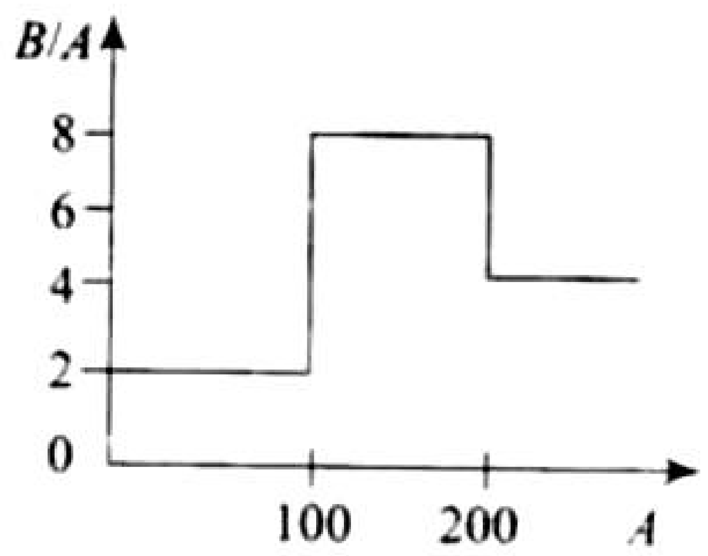
d. In nuclear fission, energy is released by fragmentation of a very heavy nucleus. **(IIT-JEE 1994)**

3. Let m_p be the mass of proton, m_n the mass of a neutron, M_1 the mass of a ${}^{20}_{10}$Ne nucleus, and M_2 the mass of a ${}^{40}_{20}$Ca nucleus. Then,

a. $M_2 = 2M_1$ b. $M_2 > 2M_1$
c. $M_2 < 2M_1$ d. $M_1 < 10(m_n + m_p)$

(IIT-JEE 1998)

4. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below:



a. Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.
b. Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.
c. Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into equal fragments.
d. Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into equal fragments. **(IIT-JEE 2008)**

Linked Comprehension Type

For Problems 1–3

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2{}_1$H, known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D–D reaction is ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + n + \text{energy}$. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1$H nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than 5×10^{14} s cm^{-3}.

It may be helpful to use the following: Boltzmann constant, $k = 8.6 \times 10^{-5}$ eV K^{-1}; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eV m.

(IIT-JEE 2009)

1. In the core of nuclear fusion reactor, the gas becomes plasma because of

a. strong nuclear force acting between the deuterons
b. Coulomb force acting between the deuterons
c. Coulomb force acting between deuteron–electron pairs
d. the high temperature maintained inside the reactor core

2. Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving toward each other, each with kinetic energy $1.5kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also, neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

a. 1.0×10^9 K $< T < 2.0 \times 10^9$ K
b. 2.0×10^9 K $< T < 3.0 \times 10^9$ K
c. 3.0×10^9 K $< T < 4.0 \times 10^9$ K
d. 4.0×10^9 K $< T < 5.0 \times 10^9$ K

3. Results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is most promising based on Lawson criterion?

a. Deuteron density $= 2.0 \times 10^{12}$ cm^{-3}, confinement time $= 5.0 \times 10^{-3}$ s
b. Deuteron density $= 8.0 \times 10^{14}$ cm^{-3}, confinement time $= 9.0 \times 10^{-1}$ s
c. Deuteron density $= 4.0 \times 10^{23}$ cm^{-3}, confinement time $= 1.0 \times 10^{-11}$ s
d. Deuteron density $= 1.0 \times 10^{24}$ cm^{-3}, confinement time $= 4.0 \times 10^{-12}$ s

For Problems 4 and 5

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e$^-$) are observed as the decay products of the neutron. Therefore, considering the decay of a netron as a two body decay process, it was predicated theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e., $n \rightarrow p + e^- + \overline{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\overline{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV.

The kinetic energy carried by the proton is only the recoil energy. **(IIT-JEE 2012)**

4. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K of the electron?

a. $0 \le K \le 0.8 \times 10^6$ eV
b. $3.0\text{ eV} \le K \le 0.8 \times 10^6$ eV
c. $3.0\text{ eV} \le K < 0.8 \times 10^6$ eV
d. $0 \le K < 0.8 \times 10^6$ eV

5. What is the maximum energy of the anti-neutrino?

a. Zero
b. Much less than 0.8×10^6 eV
c. Nearly 0.8×10^6 eV
d. Much larger than 0.8×10^6 eV

For Problems 6 and 7

The mass of nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of mass m1 and m_2 only if (m) < M. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M "only if $(m_3 + m_4) > M$". The masses of some neutral atoms are given in the table below:

(JEE Advanced 2013)

${}^{1}_{1}H$	1.007825 u	${}^{2}_{1}H$	2.014102 u	${}^{3}_{1}H$	3.016050 u	${}^{4}_{2}He$	4.002603 u
${}^{6}_{3}Li$	6.015123 u	${}^{7}_{3}Li$	7.016004 u	${}^{70}_{30}Zn$	69.925325 u	${}^{82}_{34}Se$	81.916709 u
${}^{152}_{64}Gd$	151.919803 u	${}^{206}_{82}Pb$	205.974455 u	${}^{209}_{83}Bi$	208.980388 u	${}^{210}_{84}Po$	209.982876 u

6. The correct statement is

a. The nucleus ${}^{6}_{3}Li$ can emit an alpha particle

b. The nucleus ${}^{210}_{84}Po$ can emit a proton.

c. Deuteron and alpha particle can undergo complete fusion.

d. The nuclei ${}^{70}_{30}Zn$ and ${}^{82}_{34}Se$ can undergo complete fusion.

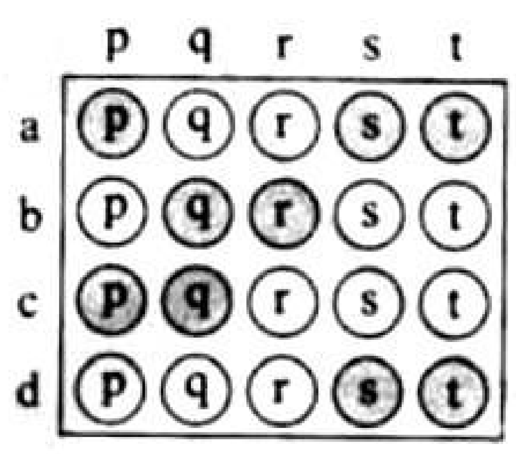
7. The kinetic energy (in keV) of the alpha particle, when the nucleus at rest undergoes alpha decay, is

a. 5319 **b.** 5422 **c.** 5707 **d.** 5818

Matching Column Type

Each question in this section contains statements given in two columns, which have to be matched. The statements in Column I are labelled a, b, c, and d, while the statements in Column II are labelled p, q, r, s and t. Any given statement in Column I can have correct matching with one or more statement(s) in Column II.

The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples. If the correct matches are a-p, a-s, and a-t; b-q and b-r; c-p and c-q; and d-s and d-t; then the correct darkening of bubbles will look like the following.



1. Match the following:

Column I	Column II
a. Nuclear fusion	**p.** Converts some matter into energy
b. Nuclear fission	**q.** Generally possible for nuclei with low atomic number
c. β-decay	**r.** Generally possible for nuclei with higher atomic number
d. Exothermic nuclear reaction	**s.** Essentially proceeds by weak nuclear forces

(IIT-JEE 2006)

2. Some laws/processes are given in Column I. Match these with the physical phenomena given in Column II.

Column I	Column II
a. Transition between two atomic energy levels	**p.** Characteristic X-rays
b. Electron emission from a material	**q.** Photoelectric effect
c. Moseley's law	**r.** Hydrogen spectrum
d. Change of photon energy into kinetic energy of electrons	**s.** β-decay

(IIT-JEE 2007)

3. Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process(es) from Column II.

Column I	Column II
a. The energy of the system is increased.	**p.** System: A capacitor, initially uncharged Process: It is connected to a battery
b. Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	**q.** System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston

c. Internal energy of the system is converted into its mechanical energy.	**r.** System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
d. Mass of the system is decreased.	**s.** System: A heavy nucleus, initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	t. System: A resistive wire loop Process: The loop is placed in a time-varying magnetic field perpendicular to its plane

(IIT-JEE 2009)

4. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

	List-I		List-II
P.	Alpha decay	1.	${}^{15}_{8}O \rightarrow {}^{15}_{7}N + ...$
Q.	$\beta +$ decay	2.	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + ...$
R	Fission	3.	${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + ...$
S	Proton-emission	4.	${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + ...$

Codes:

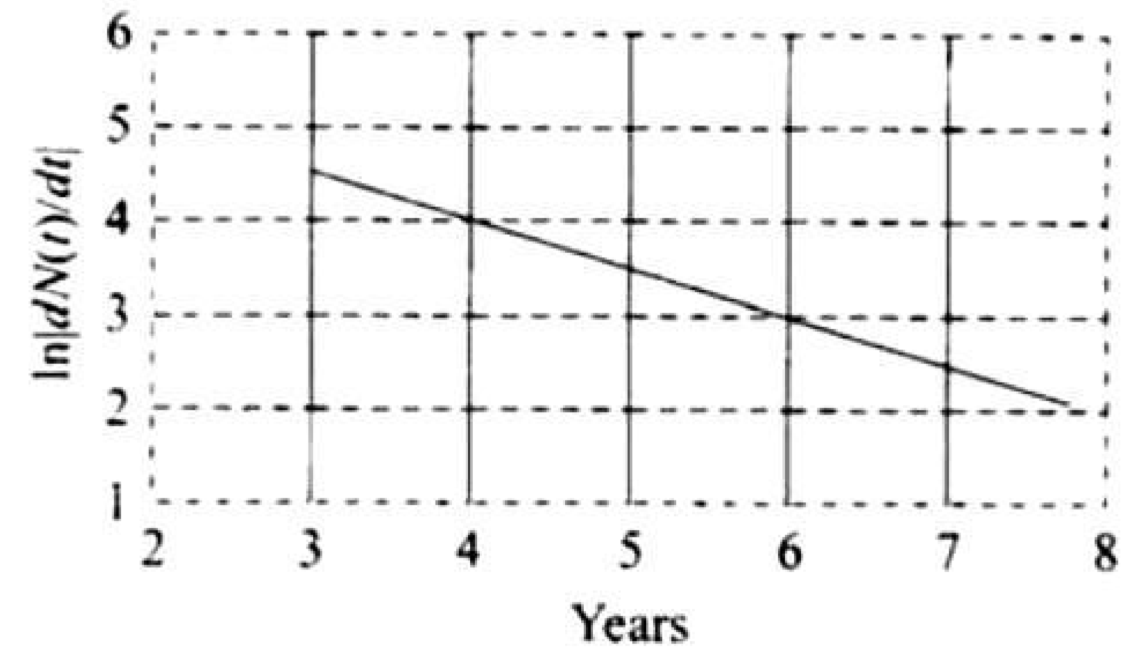
	P	Q	R	S
a.	4	2	1	3
b.	1	3	2	4
c.	2	1	4	3
d.	4	3	2	1

(JEE Advanced 2013)

Integer Answer Type

1. To determine the half life of a radioactive element, student plots a graph of $\ln|dN(t)/dt|$ versus t. Here $dN(t)/dt$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is _____.

(IIT-JEE 2010)



2. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is _____.

(IIT-JEE 2011)

3. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

(JEE Advanced 2013)

Fill in the Blanks Type

1. The radioactive decay rate of a radioactive element is found to be 10^3 disintegration s^{-1} at a certain time. If the half-life of the element is 1 s, the decay rate after 1 s is _________ and after 3 s the decay rate is _________.

(IIT-JEE 1983)

2. In the uranium radioactive series, the initial nucleus is ${}^{238}_{92}U$ and the final nucleus is ${}^{206}_{92}Pb$. When the uranium nucleus decays to lead, the number of α-particles emitted is _________ and the number of β-particles emitted is _________.

(IIT-JEE 1985)

3. When boron nucleus $({}^{10}_{5}B)$ is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element and has the mass number _________.

(IIT-JEE 1986)

4. Atoms having the same but different _________ are called isotopes. **(IIT-JEE 1986)**

5. The binding energies per nucleon for deuteron $({}_1H^2)$ and helium $({}_2He^4)$ are 1.1 MeV and 7.0 MeV, respectively. The energy released when two deuterons fuse to form a helium nucleus $({}_2He^4)$ is _________. **(IIT-JEE 1988)**

6. In the nuclear process ${}_6C^{11} \rightarrow {}_5B^{11} + \beta^+ + X$, X stands for _________.

(IIT-JEE 1992)

7. Consider the following reaction:

$${}^2_1H + {}^2_1H = {}^4_2He + Q$$

[Mass of the deuterium atom = 2.0141 u and mass of helium atom = 4.0024]

This is a nuclear _________ reaction in which the energy Q released is MeV. **(IIT-JEE 1996)**

True/False Type

1. The order of magnitude of the density of nuclear matter is 10^4 kg m^{-2}. **(IIT-JEE 1989)**

Subjective Type

1. How many electrons, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24? **(IIT-JEE 1982)**
2. An uranium nucleus (atomic number = 92 and mass number = 231) emits an α-particle and the resultant nucleus emits a β-particle. What are the atomic and mass numbers of the final nucleus? **(IIT-JEE 1982)**
3. There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? Given mass of neutron = 1.676×10^{-27} kg. **(IIT-JEE 1986)**
4. It is proposed to use the nuclear fusion reaction $^2_1H + ^2_1H \rightarrow ^4_2He$ in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day. (The masses of ^2_1H and ^4_2He are 2.0141 atomic mass units and 4.0026 atomic mass units respectively). **(IIT-JEE 1990)**
5. A nucleus X, initially at rest, undergoes alpha decay according to the equation.

$$^A_{92}X \rightarrow ^{228}_{Z}Y + \alpha$$

   a. Find the values of A and Z in the above process.
   b. The alpha particle produced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 Tesla. Find the energy (In MeV) released during the process and the binding energy of the parent nucleus X.

   Given that: $m(Y) = 238.03\ u$; $m(^1_0n) = 1.009\ u$

$$m(^4_2He) = 4.003\ u; m(^1_1H) = 1.008\ u$$

(IIT-JEE 1991)

6. A small quantity of solution containing ^{24}Na radionuclide (half-life 15 h) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 h shows an activity of 296 disintegrations min^{-1}. Determine the total volume of blood in the body of the person. Assume that the radioactivity solution mixes uniformly in the blood of the person. (1 curie = 3.7×10^{10} disintegrations s^{-1}.) **(IIT-JEE 1994)**
7. At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 s, the number of undecayed nuclei reduces to 12.5%. Calculate (a) mean life of the nuclei and (b) the time in which the number of decayed nuclei will further reduce to 6.25% of the reduced number. **(IIT-JEE 1996)**
8. The element curium $^{218}_{96}$Cm has a mean-life of 10^{13} s. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV energy. The masses involved in α-decay are as follows: $^{218}_{98}$Cm = 248.07220 u, $^{244}_{94}$Pu = 244.064100 u, and $^4_2$He = 4.002603 u.
   Calculate the power output from a sample of 10^{20} Cm atoms. (1 u = 931 MeV c^{-2}) **(IIT-JEE 1997)**
9. Nuclei of radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t = 0$, there are N_0 nuclei of the element.
   a. Calculate the number N of nuclei of A at time t.
   b. If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life time of A and also the limiting value of N at $t \rightarrow \infty$. **(IIT-JEE 1998)**
10. In a nuclear reactor, ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required. **(IIT-JEE 2001)**
11. A nucleus at rest undergoes α-decay emitting an α-particle of de Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the α-particle is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of parent nucleus in amu (1 amu = 931.470 MeV c^{-2}) **(IIT-JEE 2001)**
12. A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x = 0.1$ s^{-1}. Y further decays to a stable nucleus Z with a decay constant $\lambda_y = 1/30$ s^{-1}. Initially, there are only X nuclei and their number is $N_0 = 10^{20}$. Set-up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by $N_y(t)\ \{N_0\lambda_x/(\lambda_x - \lambda_y)\}\ [\exp(-\lambda_y t) - \exp(-\lambda_x t)]$. Find the time at which N_Y is maximum and determine the populations X and Z at that instant. **(IIT-JEE 2001)**
13. A radioactive element decays by β-emission. A detector records n beta particles in 2 s and in next 2 s, it records $0.75n$ beta particles. Find mean life correct to nearest whole number. Given ln 2 = 0.6931, ln 3 = 1.0986. **(IIT-JEE 2003)**
14. A rock is 1.5×10^9 years old. The rock contains ^{238}U which disintegrates to form ^{206}U. Assume that there was no ^{206}Pb in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of ^{238}U to that of ^{206}Pb in the rock. Half-life of ^{238}U is 4.5×10^9 years. ($2^{1/3} = 1.259$) **(IIT-JEE 2004)**
15. A radioactive sample of ^{238}U decays to Pb through a process for which the half-life is 4.5×10^9 years. Find the ratio of number of nuclei of Pb to ^{238}U after a time of 1.5×10^9 years. Given $(2)^{1/3} = 1.26$. **(IIT-JEE 2004)**

16. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. Find (a) atomic number of nucleus. (b) The frequency of K_α line of the X-ray produced. ($R = 1.1 \times 10^7$ m^{-1} and $c = 3 \times 10^8$ m/s). **(IIT-JEE 2005)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. b.	**2.** c.	**3.** c.	**4.** c.	**5.** d.
6. a.	**7.** b.	**8.** b.	**9.** c.	**10.** b.
11. c.	**12.** d.	**13.** d.	**14.** b.	**15.** b.
16. b.	**17.** c.	**18.** c.	**19.** a.	**20.** d.
21. c.	**22.** d.	**23.** a.	**24.** c.	**25.** a.
26. b.	**27.** b	**28.** c.	**29.** b.	**30.** a.
31. a.				

Multiple Correct Answers Type

1. a., b., c. **2.** a., d. **3.** c., d. **4.** b., d.

Linked Comprehension Type

1. d. **2.** a. **3.** b. **4.** d. **5.** c.
6. c. **7.** a.

Matching Column Type

1. a. → p., q.; b. → p., r.; c. → p., s.; d. → p., q., r.
2. a. → p., r.; b. → q., s.; c. → p.; d. → q.
3. a. → p., q., t.; b. → q.; c. → s.; d. → s.
4. P. → 2.; Q. → 1.; R. → 4.; S. → 3.

Integer Answer Type

1. (8) **2.** (1) **3.** (4)

Fill in the Blanks Type

1. 500 dps, 125 dps **2.** eight, six
3. lithium, 7 **4.** atomic number, mass number
5. 23.6 MeV **6.** Neutrino **7.** Fusion, 24

True/False Type

1. False

Subjective Type

1. zero, 11, 13 **2.** 91, 234
3. 3.96×10^{-6} **4.** 120.26 kg
5. (a) 90, 232 (b) 5.34 Me V, 1823.2 MeV
6. 5.95 L **7.** (a) 14.43 s (b) 40 s **8.** 3.32×10^{-5} W
9. (a) $\frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}]$ (b) (i) $\frac{3}{2}N_0$ (ii) $2N_0$
10. 3.847×10^4 kg
11. (a) 6.25 MeV (b) 227.62 amu
12. (a) $\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X, \left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y$

and $\left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y N$

(b) 16.48 s

(c) $N_X = 1.92 \times 10^{19}$, $N_Z = 2.32 \times 10^{19}$
13. (a) $Z = 3$ (b) 4052.3 nm **14.** 3.861
15. 0.26 **16.** (a) 26 (b) 1.546×10^{18} Hz

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. b. $T_{1/2} = 3.8$ day

$$\therefore \quad \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{3.8} = 0.182$$

If the initial number of atoms is $a = A_0$, then after time t the number of atoms is $a/20 = A$. We have to find t.

$$t = \frac{2.303}{\lambda}\log\frac{A_0}{A} = \frac{2.303}{0.182}\log\frac{a}{a/20} = \frac{2.303}{0.182}\log 20$$
$$= 16.46 \text{ day}$$

2. c. β-particles are radioactive material emitted by the nucleus.

3. c. $4\,{}_1^1H^+ \rightarrow {}_2^4He^{2+} + 2e^- + 26$ MeV

represents a fusion reaction.

4. c. Following nuclear reaction takes place

${}_0n^1 \Rightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu}$

$\bar{\nu}$ is antineutrino.

5. d. In nuclear fusion two light nuclei combine to give a heavier nucleus and possibly other product.

6. a. At room temperature the energy of air molecules is approximately 2×10^{-2} eV. The binding energy of heavy nuclei per nucleon is in the range of 6.5–7.5 MeV. The X-ray photon has energy in the rate 10 keV and the photon energy of visible light is in the range 1–2 eV.

7. b. From $R = R_0\left(\frac{1}{2}\right)^n$, we have

$$1 = 64\left(\frac{1}{2}\right)^n$$

or $\quad n = 6$ = number of half-lives

$t = n \times t_{1/2} = 6 \times 2 = 12$ h

8. b. Half-life $T = \frac{0.6931}{\lambda} = \frac{\ln 2}{\lambda}$

Mean-life $\tau = \frac{1}{\lambda}$

9. c. The given reactions are

$${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + p$$
$${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$$
$$3\,{}_1H^2 \rightarrow {}_2He^4 + n + p$$

Mass defect,

$$\Delta m = (3 \times 2.014 - 4.001 - 1.007 - 1.008) \text{ a.m.u.}$$
$$= 0.026 \text{ a.m.u.}$$

Energy released $= 0.026 \times 931$ MeV

$= 0.026 \times 931 \times 1.6 \times 10^{-13}$ J

$= 3.87 \times 10^{-12}$ J

This is the energy produced by the consumption of three deuteron atoms. Therefore, total energy released by 10^{40} deuterons is

$$\frac{10^{40}}{3} \times 3.87 \times 10^{-12} \text{ J} = 1.29 \times 10^{28} \text{ J}$$

The average power radiated is $P = 10^6$ W or 10^{16} J s^{-1}.

Therefore, total time to exhaust all deuterons of the star will be

$$t = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12} \text{ s} \approx 10^{12} \text{ s}$$

10. b. Fast neutrons can be easily slowed down by passing them through water. This is because of comparable masses the energy passed by neutron to water molecule is high.

11. c. The penetrating power is dependent on velocity. For a given energy, the velocity of γ-radiation is highest and α-particle is least.

12. d. The mass defect for ^{64}Zn is more than that for ^{64}Cu. So, Zn is more stable. Therefore, ^{64}Cu is radioactive and will decay to ^{64}Zn through β^--decay as follows

$${}_{64}^{29}Cu \rightarrow {}_{30}^{64}Zn + {}_{-1}^{0}e$$

Alternative solution:

By the conservation of charge and nucleons, only potential is feasible.

13. d. Number of nuclei decreases exponentially.

$$N = N_0 e^{-\lambda t}$$

Rate of decay, $-\frac{dN}{dt} = \lambda N$

Therefore, decay process lasts up to $t = \infty$. Therefore, a given nucleus may decay at any time after $t = 0$.

14. b. According to Doppler's effect of light, the wavelength shift is given by

$$\Delta\lambda = \frac{V}{c} \times \lambda$$

$$\Rightarrow \quad V = \frac{\Delta\lambda \times c}{\lambda} = \frac{(706 - 656)}{656} \times 3 \times 10^8$$
$$= 2 \times 10^7 \text{ m s}^{-1}$$

15. b. Nuclear density of an atom of mass number A,

$$D = \frac{\text{mass}}{\text{volume}} = \frac{A(1.67 \times 10^{-27})}{\frac{4}{3}\pi[1.25 \times 10^{-15} A^{1/3}]^3}$$

$$\left[\because V = \frac{4}{3}\pi R^3, R = R_0 A^{1/3}, R_0 = 1.25 \times 10^{-15}\right]$$

$$\therefore \quad D = 2 \times 10^{17} \text{ kg m}^{-3}$$

16. b. ${}_{10}^{22}Ne \rightarrow {}_2^4He + {}_2^4He + {}_6^{14}X$

The new element X has a atomic number 6. Therefore, the element is carbon.

17. c. Energy will be released when stability increases. This will happen when binding energy per nucleon increases.

	Reactant	Product
Reaction (a)	60×8.5 MeV = 510 MeV	$20 \times 30 \times 5$ = 300 MeV
Reaction (b)	120×7.5 = 900 MeV	$(90 \times 8 + 30 \times 5)$ = 870 MeV
Reaction (c)	120×7.5 = 900 MeV	$2 \times 60 \times 8.5$ = 1020 MeV
Reaction (d)	90×8 = 720 MeV	$(60 \times 8.5 + 30 \times 5)$ = 600 MeV

18. c. $(t_{1/2})_x = (t_{\text{mean}})_y$

$$\frac{0.693}{\lambda_x} = \frac{1}{\lambda_y}$$
$$\lambda_x = 0.693\,\lambda_y$$
$$\lambda_x < \lambda_y$$

or Rate of decay = λN

Inititally, number of atoms (N) of both are equal but since $\lambda_y > \lambda_x$, therefore, Y will decay at a faster rate than X.

19. a. Both the beta rays and the cathode rays are made up of electrons. So, only option (a) is correct.

Gamma rays are electromagnetic waves.

Alpha particles are doubly ionized helium atoms.

Protons and neutrons have approximately the same mass.

Therefore, (b), (c) and (d) are wrong options.

20. d. $N_1 = N_0 e^{-10\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$$\therefore \quad \frac{N_1}{N_2} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = \frac{1}{e^{9\lambda t}}$$

Given,

$$\frac{N_1}{N_2} = \frac{1}{e} \Rightarrow \frac{1}{e^{9\lambda t}} = \frac{1}{e}$$

or $9\lambda t = 1$ or $t = \left(\frac{1}{9\lambda}\right)$

21. c. We know that in a nucleus, neutron converts into proton as follows:

$$n \rightarrow p^+ + e^-$$

Thus, decay of neutron is responsible for β-radiation orgination.

22. d. $N_1 = N_0 e^{-\frac{t}{\tau}}$ (i) and $\tau = \frac{1}{\lambda_1}$

$N_2 = N_0 e^{-\lambda_2 t} = N_0 e^{-\frac{1}{5\tau}}$ (ii) and $5\tau = \frac{1}{\lambda_2}$

Adding (i) and (ii), we get

$$N = N_1 + N_2 = N_0 (e^{-t/\tau} + e^{-t/5\tau})$$

(a) is not the correct option as there is a time τ for which N is constant, which means for time τ there is no process of radioactivity which does not makes sense.

(b) and (c) show intermediate increase in the number of radioactive atoms which is impossible as N will only decrease exponentially. Hence, the correct option is (d).

23. a. $R = R_0 \left(\frac{1}{2}\right)^n$ (i)

Here R = activity of radioactive substance after n half-lives

$= \frac{R_0}{16}$ (given)

Substituting in equation (i), we get $n = 4$

$\therefore \quad t = (n)t_{1/2} = (4)(100\ \mu s) = 400\ \mu s$

24. c. During γ-decay atomic number (Z) and mass number (A) does not change. So, the correct option is (c) because in all other options either Z, A or both is/are changing.

25. a. We know that radius of the nucleus,

$$R = R_0 A^{1/3}$$

where A is the mass number.

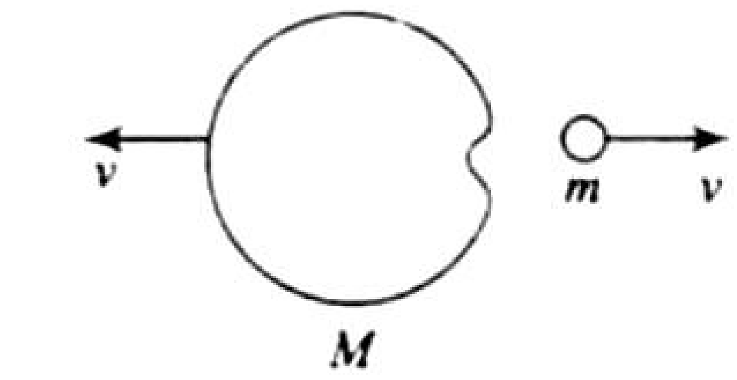
$$\therefore \quad R^3 = R_0^3 A \Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

$\Rightarrow$ Volume $\propto$ mass

26. b. By conservation of linear momentum,

$$MV = mv \Rightarrow 216\ V = 4v \Rightarrow V = \frac{v}{54} \quad \text{(i)}$$

By energy conservation,



$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = 5.5 \times 1.6 \times 10^{-13}\ \text{J}$$

$$\Rightarrow 4 \times 1.67 \times 10^{-27} v^2 + 216 \times 1.67 \times 10^{-27} \times \frac{v^2}{54 \times 54} = 2 \times 5.5 \times 1.6 \times 10^{-13}$$

$$\Rightarrow 1.67 \times 10^{-27} v^2 \left[4 + \frac{216}{54 \times 54}\right] = 2 \times 5.5 \times 1.6 \times 10^{-13}$$

$$\text{or} \quad v^2 = \frac{2 \times 5.5 \times 1.6 \times 54 \times 54 \times 10^{-13}}{(4 \times 54 \times 54 + 216)1.67 \times 10^{-27}} = 2.586 \times 10^{14}$$

$$\therefore \text{KE of } \alpha\text{-particle} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 4 \times 1.67 \times 10^{-27} \times 2.586 \times 10^{14}$$

$$= 8.637 \times 10^{-13}\ \text{J} = \frac{8.637 \times 10^{-13}}{1.6 \times 10^{-13}}\ \text{MeV}$$

$$= 5.4\ \text{MeV}$$

Alternative solution:

By conservation of momentum,

$$P_1 = P$$

$$\Rightarrow \sqrt{2k_2 m_1} = \sqrt{2k_2 m_2}$$

$$\Rightarrow \sqrt{2k_1(216)} = \sqrt{2k_2(4)}$$

$$\Rightarrow k_2 = 54\ k_1 \quad \text{(i)}$$

Also, $k_1 + k_2 = 5.5$ MeV (ii)

Solve Eqs. (i) and (ii).

27. b. We know that $\lambda = \frac{2.303}{t}\log\frac{A_0}{A}$, where A_0 is the initial activity. A is the activity at time t.

$$\therefore \quad \lambda = \frac{2.303}{280}\log\frac{A_0}{6000} = \frac{2.303}{420}\log\frac{A_0}{3000}$$

On solving, we get

$$A_0 = 24000\ \text{dps}$$

28. c. ${}^4_2\text{He} \rightarrow {}^{16}_8\text{O}$

$$\text{BE} = \Delta m \times 931.5\ \text{MeV}$$
$$= (4 \times 4.0026 - 15.9994) \times 931.5$$
$$= 10.24\ \text{MeV}$$

29. b. Radioactive decay is a random process. Each decay is a completely independent event. Therefore, which particular nucleus will decay at a given instant of time cannot be predicted. In other words, when a particular nucleus will decay cannot be predicted. Each nucleus has same probability of disintergration.

30. a. Iodine and Yttrium are medium sized nuclei and therefore have more binding energy per nucleon as compared to uranium which has a big nuclei and less BE/nucleon. In other words, Iodine and Yttrium are more stable and therefore possess less energy and less rest mass. Also, when uranium nuclei explodes, it will convert into I and Y nuclei having kinetic energies.

31. a. Activity of $S_1 = \frac{1}{2}$ (activity of S_2)

or $\lambda_1 N_1 = \frac{1}{2}(\lambda^2 N^2)$

$$\frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$$

$$\frac{T_1}{T_2} = \frac{2N_1}{N_2} \qquad \left(T = \text{half-life} = \frac{\ln 2}{\lambda}\right)$$

Given $N_1 = 2N_2$

$$\frac{T_1}{T_2} = 4$$

Multiple Correct Answer Type

1. **a., b., c.** Nuclear fusion reaction occurs when two or more lighter nuclei combine to produce a heavier nucleus.
2. **a., d.** In nuclear fusion, two or more lighter nuclei are combined to form a relatively heavy nucieus and thus, releasing the energy.
3. **c., d.** Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.
   $^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons.
   Therefore, mass of $^{20}_{10}Ne$ nucleus,
   $M_1 < 10(m_p + m_n)$
   Also, heavier the nucleus, more is the mass defect.
   $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$
   Thus, $10(m_n + m_p) > M_2 - M_1$
   or $M_2 < M_1 + 10(m_p + m_n)$
   Now, since
   $M_1 < 10(m_p + m_n)$
   $\therefore \quad M_2 < 2M_1$
4. **b., d.** In fusion, two or more lighter nuclei combine to make a comparatively heavier nucleus.
   In fission, a heavy nucleus beaks into two or more comparatively lighter nuclei.
   Further, energy will be released in a nuclei process if total binding energy increases.
   Hence, correct options are (b) and (d).

Linked Comprehension Type

1. d. The very high temperature maintained in the plasma causes fusion.

2. a. Given $k = 8.6 \times 10^{-3}$ eV/K

$$\frac{e}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}\text{ eVm}$$

K.E. of two deuterons = 2 × 1.5 kT = 3 kT

This K.E has to overcome the potential energy of the nucleus.

$$\Rightarrow \quad 3kT = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$T = \frac{1}{k}\cdot\frac{1}{3\times r}\cdot\frac{e^2}{4\pi\varepsilon_0} \therefore\ T = \frac{1.44\times10^{-9}}{8.6\times10^{-5}}\times\frac{1}{3\times4\times10^{-15}}$$

$\Rightarrow \quad T = 1.39 \times 10^9$ K

3. b. The product of the deuteron density and confinement time t_0 should be greater than Lawson criteria, 5×10^{14} s cm^{-3}.

deuteron density 8.0×10^{14} cm^{-3}

confinement time 9.0×10^{-1} s

$\therefore$ Lawson criteria $= 8.0 \times 10^{14} \times 9.0 \times 10^{-1} = 7.2 \times 10^{14}$

It is not enough if the density is very high but the confinement time also should be high enough for the reaction.

4. d. Totai energy remains conserved. Energy is shared by antineutrino, proton and electron. Kinetic energy of electron has continuous spectrum and it is maximum when antineutrino does not share any kinetic energy. So total energy is shared with proton and electron only.

$\therefore \quad K \le 0.8 \times 10^6$ eV

and kinetic energy of electron will be minimum or zero when total energy is shared by proton and antineutrino.

$\therefore \quad 0 \le K \le 0.8 \times 10^6$ eV

5. c. $K_p + K_e + K_\nu = 0.8\times10^6$ eV

When electron has zero kinetic energy is shared by antineutrino and proton.

Then, $K_p + K_\nu = 0.8\times10^6$ eV

As antineutrino is very high mass in comparison to proton so it will have almost contribution in total energy.

$\therefore$ Its energy is almost 0.8×10^6 eV

6. c.

$$^6_3\text{Li} \to {}^4_2\text{He} + {}^2_1\text{H}$$

$$\frac{Q}{C^2} = 6.015123 - 4.002603 - 2.014102$$

$$0 = -0.001582 < 0$$

So no α-decay is possible

$$^{210}_{84}\text{P}_0 \to {}^1_1\text{H} + {}^{209}_{83}\text{Bi}$$

$$\frac{Q}{C^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0$$

So, this reaction is not possible

$$^2_1\text{He} + {}^4_2\text{He} \to {}^6_3\text{Li}$$

$$\frac{Q}{C^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$$

So, this reaction is possible

$$^{70}_{30}\text{Zn} + {}^{82}_{34}\text{Se} \to {}^{152}_{64}\text{Gd}$$

$$\frac{Q}{C^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$$

So this reaction is not possible

7. a.

$$^{210}_{84}\text{Po} \to {}^4_2\text{He} + {}^{206}_{82}\text{Pb}$$

$$Q = (209.982876 - 4.002603 - 205.97455)C = 5.422\text{ MeV}$$

From conservation of momentum,

$$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$$

$$\therefore \quad K_1 = \frac{103}{2}K_2$$

$$K_1 + K_2 = 5.422$$

$$K_2 + \frac{2}{103}K_1 = 5.422$$

$$\Rightarrow \quad \frac{105}{103}K_1 = 5.422$$

$$\therefore \quad K_1 = 5.319 \text{ MeV} = 5319 \text{ keV}$$

Matching Column Type

1. a → p, q; b → p, r; c → p, s; d → p, q, r.

In a nuclear fusion reaction, matter is converted into energy and nuclei low atomic number generally give this reaction.

In a nuclear fission reaction, matter is converted into energy and nuclei of high atomic number generally give this reaction.

2. a → p, r; b → q, s; c → p; d → q.

Characteristic X-rays are produced due to transition of electrons from one energy level to another.

Similarly, the lines in the hydrogen spectrum are obtained due to transition of electrons from one energy level to another.

In photoelectric effect electrons from the metal surface are emitted out upon the incidence of light of appropriate frequency. In β-decay, electrons are emitted from the nucleus of an atom.

Moseley gave a law which related frequency of emitted X-rays with the atomic number of the target material as $\sqrt{v} = a(z - b)$.

In photoelectric effect, energy of photons of incident ray gets converted into kinetic energy of emitted electrons.

3. a → p, q, t; b → q; c → s; d → s.

(a) → (p):

The energy of a capacitor is increased when connected to a battery.

(a) → (q):

By compressing a gas adiabatically, the internal energy increases.

(a) → (t):

By placing a loop of wire in a time varying magnetic field perpendicular to its plane $\varepsilon = -\left(\frac{d\phi}{dt}\right)$ an e.m.f. is generated. This increases the energy of the loop.

(b) → (q):

Mechanical energy of pushing the piston, does work on a gas, increasing its energy by increasing the velocity of random motion.

(c) → (s):

Internal energy of the system is converted into mechanical energy. Energy internal to system, is converted to energy of motion, temperature and photons in nuclear fission (natural radioactivity) of heavy fragments.

(d) → (s):

The increase in the energy of the products of fission or radioactivity comes from a decrease in the initial mass.

4. c.

P. → 2.; Q. → 1.; R. → 4.; S. → 3.

$^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{1}\beta$ (beta decay)

$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$ (alpha decay)

$^{185}_{83}Bi \rightarrow ^{184}_{82}Pb + ^{1}_{1}H$ (proton emission)

$^{239}_{94}Ph \rightarrow ^{140}_{57}La + ^{99}_{37}Rb$ (fission)

Integer Answer Type

1. (8) $N = N_0 e^{-\lambda t}$

$\ln|dN/dt| = \ln(N_0\lambda) - \lambda t$

From graph

$$\lambda = \frac{1}{2} \text{ per year}$$

$$\frac{t_1}{2} = \frac{0.693}{1/2} = 1.386 \text{ year}$$

$$4.16 \text{ years} = 3t_{1/2}$$

$$p = 8$$

2. (1) $N = N_0 e^{-\lambda t}$

$$\frac{dN}{dt} = 10^{10} = N_0(\lambda)e^{-10^{-9}t}$$

At $(t = 0)$

$10^{10} = N_0 10^{-9}$

Mass of sample $= N_0 = 10^{-25}$

$= N_0$ (mass of the atom)

$= 10^{-6}$ kg

$= 10^{-6} \times 10^3$ g

$= 10^{-3}$ g

$= 1$ mg

3. (4) $f = (1 - e^{-\lambda t}) = 1 - e^{-\lambda t} \approx 1 - (1 - \lambda t) = \lambda t$

$f = 0.04$

Hence % decay $\approx 4\%$

Fill in the Blanks Type

1. $\frac{dN}{dt} = 1000$ at $t = 0$ s

$$t_{1/2} = 1 \text{ s} \Rightarrow \lambda = 0.693 \text{ s}^{-1}$$

At $t = 1$ s,

$$1 = \frac{2.303}{0.693}\log 10 \frac{N_0}{N}$$

Now,

$$t = \frac{2.303}{\lambda}\log_{10}\frac{N_0}{N}$$

$$\therefore \quad 0.3010 = \log_{10}\frac{N_0}{N} \Rightarrow \log_{10}\frac{N_0}{N} = \text{lc}$$

$$\therefore \quad \frac{N_0}{N} = 2 \Rightarrow N = \frac{N_0}{N}$$

$$\therefore \quad N = \frac{1000}{2} = 500$$

Here,

$$\frac{dN}{dt} \propto N_0$$

and $\frac{dN}{dt} \propto N$

$$\therefore \quad \frac{dN}{dt} = 500 \text{ dps} \quad \text{at} \quad t = 1 \text{ s}$$

Similarly, at $t = 3$ s, $\frac{dN}{dt} = 125$ dps.

Alternative solution:

$$A = A_0 \left(\frac{1}{2}\right)^n$$

where A_0 = initial activity = 100 dps (given), A = activity after n half-lives.

$$\text{At } t = 1, n = 1 \Rightarrow A = 1000\left(\frac{1}{2}\right)^1 = 500 \text{ dps}$$

$$\text{At } t = 3, n = 3 \Rightarrow A = 1000\left(\frac{1}{2}\right)^3 = 125 \text{ dps}$$

2. $^{238}_{92}U \rightarrow ^{206}_{82}Pb + x\,^4_2He + y\,^0_{-1}e$

First, we find the number of α-particles. The change in mass number during the decay from uranium to lead is $238 - 206 = 32$. Therefore, the number of α-particles (with mass no. 4) is $32/4 = 8$.

The change in atomic number (i.e., number of protons) taking place when 8 α-particles are emitted and lead is formed $= 92 - (82 + 16) = 6$ which means six β-particles are emitted.

3. $^{10}_5B + ^1_0n \rightarrow ^4_2He + ^7_3Li$

The resulting nucleus is of element lithium and mass number is 7.

4. Atomic number, mass number

5. $^2_1H + ^2_1H \rightarrow ^4_2He$

Binding energy of two deuterons is

$$2[1.10020 \times 2] = 4.4 \text{ MeV}$$

Binding energy of one helium = $4 \times 7 = 2.8$ MeV

The energy released = $28 - 4.4 = 23.6$ MeV

6. $^{11}_6C \rightarrow ^{11}_5B + \beta^+ + X \Rightarrow ^{11}_6C \rightarrow ^{11}_5B + ^0_{+1}e + X$

(neutrino)

The balancing of atomic number and mass number is OK. Therefore X stands for energy.

7. This is a nuclear fusion reaction.

Energy released = (Δm) [931.5 MeV/u)

$$= [4.0024 - 2 \times 2.0141] \times 931.5 \text{ MeV}$$

$$= -24.03 \text{ MeV (heat released)}$$

True/False Type

1. False.

$$\text{Density} = \frac{m}{V} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi\left[R_0 A^{1/3}\right]} = 3 \times 10^{17} \text{ kg m}^{-3}$$

where A = mass number

$$= \frac{1.67 \times 10^{-27}}{1.33 \times 3.14 \times (1.1 \times 10^{-15})}$$

Alternative solution:

Remember that the order of nuclear density is 10^{17} kg m^{-3}.

Subjective Type

1. Number of protons in given nucleus

= its atomic number = 11

Number of electrons in an atom

= Number of protons = 11

Number of neutrons = mass number (A) – atomic number (Z)

= 24 – 11 = 13

2. An uranium nucleus (atomic number = 92 and mass number = 231) emits an α-particle, we have the product nucleus X

$$_{92}U^{238} \rightarrow {}_{90}X^{234} + {}_2He^4$$

When the product nucleus emits a β-particles, the final nucleus is Y (say) given by $_{92}X^{234} \rightarrow {}_{91}Y^{234} + {}_{-1}\beta^0$.

Thus atomic number of final nucleus = 91, mass number of final nucleus = 234.

3. The velocity v of neutron is given by

$$\frac{1}{2}mv^2 = 0.0327 \text{ eV}$$

or $\frac{1}{2} \times 1.675 \times 10^{-27} v^2 = 0.0327 \times 1.6 \times 10^{-19}$ J

$$\therefore \quad v^2 = \frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}$$

or $v = 2.5 \times 10^3$ m s^{-1}

Time taken by neutron to travel a distance of 10 m is

$$t = \frac{\text{disance}}{\text{velocity}} = \frac{10}{2.5 \times 10^3} = 4 \times 10^{-3} \text{ s}$$

and half-life $T = 700$ s.

The fraction of neutrons decayed in time t is

$$\frac{N}{N_0} = e^{-\lambda t} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{4 \times 10^{-3}/700} = \left(\frac{1}{2}\right)5.7 \times 10^{-6}$$

$$= 0.999952$$

Fraction of neutrons decayed is

$$1 - 0.999952 = 0.000048 = 4.8 \times 10^{-5}$$

4. Mass defect in the given nuclear reaction:

Δm = 2 (mass of deuterium) – (mass of helium)

= 2(2.0141) – (4.0026) = 0.0256

Hence, energy released in the reaction

$\Delta E = (\Delta m)$ (931 .48) MeV = 23.85 MeV

$= 23.85 \times 1.6 \times 10^{-13}$ J

$= 3.82 \times 10^{-12}$ J

Efficiency of reactor is only 25%, therefore, 25 % of

$$\Delta E = \left(\frac{25}{100}\right)(3.82 \times 10^{-12}) \text{ J} = 9.55 \times 10^{-13} \text{ J}$$

It means by the fusion of two deuterium nuclei, 9.55×10^{-13} J energy is available to the nuclear reactor.

Hence total energy required in one day to run the reactor with a given power of 200 MW:

$$E_{Total} = 200 \times 10^6 \times 24 \times 3600 = 1.728 \times 10^{23} \text{ J}$$

$\therefore$ Total number of deuterium nuclei required for this purpose:

$$n = \frac{E_{Total}}{\Delta E/2} = \frac{2 \times 1.728 \times 10^{13}}{9.55 \times 10^{-13}} = 0.362 \times 10^{26}$$

$\therefore$ Mass of deuterium required

= (Number of g-moles of deuterium required) × 2 g

$$= \left(\frac{0.362 \times 10^{26}}{6.02 \times 10^{23}}\right) \times 2 = 120.26 \text{ kg}$$

5. We are given the equation

$$^{A}_{92}X \rightarrow ^{228}_{Z}Y + \alpha \quad \text{(i)}$$

We know the α-particle is a helium nucleus ^4_2He. Therefore we can write equation

$$^{A}_{92}X \rightarrow ^{228}_{Z}Y + ^{4}_{2}He \quad \text{(ii)}$$

a. Now using the principle of conservation of charge, we have

$92 = Z + 2$ or $Z = 90$

From the principle of conservation of mass we have

$A = 228 + 4$ or $A = 232$

b. Let v and V and m and M respectively denote the velocities and masses of the alpha particle and the resulting nucleus Y after the decay respectively. The conservation of linear Momentum principle gives (since the initial momentum is zero)

$MV + mv = 0$ or $|mv| = |MV|$

$$|v| = \left|\frac{MV}{m}\right| \quad \text{(iii)}$$

Let r be the radius of the circular path of alpha particle in magnetic field B, we can write

$$\frac{mv^2}{r} = qvB \quad \text{or} \quad v = \frac{qBr}{m} = \frac{2eBr}{m} \quad \text{(iv)}$$

Here $m = 4.003 \times (1.66 \times 10^{-27})$ kg, substituting the given values in (iv) and solving we get $v = 1.59 \times 10^7$ ms^{-1}.

The total energy released in this process:

$E = KE$ of alpha particle $+ KE$ of nucleus Y

$$= \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Using (iii), we have $E = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$

$$\Rightarrow \quad E = \frac{1}{2}mv^2\left(1 + \frac{m}{M}\right)$$

Substituting the given values, we get $E = 5.34$ MeV.

6. Initial activity of ^{24}Na.

$$A = \frac{dN}{dt} = 1.0\,\mu\text{C}$$

$= 1.0 \times 10^{-6} \times 3.7 \times 10^{10}$

$= 3.7 \times 10^4$ disintegrations s^{-1}

Half-life, $T = 15$ h $= 15 \times 3600$ s

Initial activity,

$$A = \frac{dN}{dt} = \lambda N_0$$

$$3.7 \times 10^4 = \frac{0.693}{15 \times 3600} N_0$$

$$\therefore \quad N_0 = \frac{3.7 \times 10^4 \times 15 \times 3600}{0.693}$$

$= 2.883 \times 10^9$

Let the number of radioactive nuclei present after 5 h be N' in 1 cm^3 of sample of blood. Then,

$$\frac{dN}{dt} = \lambda N' \quad \Rightarrow \quad \frac{dN}{dt}\frac{296}{60} = \frac{0.693}{15 \times 3600} N'$$

$$\therefore \quad N' = \frac{296 \times 15 \times 3600}{60 \times 0.693} = 3.844 \times 10^5$$

If N_0' is initial number of radioactive nuclei in 1 cm^3 of sample, then

$$\frac{N'}{N_0'} = \left(\frac{1}{2}\right)^{t/T}$$

$N_0' = (2)^{t/T} N'$

$= (2)^{5/15} N' = (2)^{1/3} \times 3.844 \times 10^5$

Let $y = (2)^{1/3}$.

$$\therefore \quad \log y = \frac{1}{3}\log 2 = \frac{1}{3} \times 0.3010 = 0.1003$$

$\Rightarrow \quad y = \text{Antilog } 0.1003 = 1.2598$

$\therefore \quad N_0' = 1.2598 \times 3.844 \times 10^5$

$= 4.843 \times 10^5$

$$\text{Volume of blood} = \frac{N_0}{N_0'} = \frac{2.883 \times 10^9}{4.843 \times 10^5} = 0.595 \times 10^4 \text{ cm}^3$$

$= 5.95$ L

7. Half-life of radioactive sample, i.e., the time in which number of undecayed nuclei becomes half (T) is 10 s.

a. Mean life, $\tau = \frac{T}{\log_e 2} = \frac{10}{0.693}\text{s} = 1.443 \times 10 = 14.43$ s

b. The reduced number further reduces to 6.25% in n half-lives given by

$$\frac{N}{100} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{6.25}{100} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

Time, $t = 4T = 4 \times 10 = 40$ s

8. Activity (or rate) of fission,

$$A = \frac{dN}{dt} = \lambda N = \frac{1}{\tau}N = \frac{10^{20}}{10^{13}} = 10^7$$

As probability of fission is 8% only, therefore, actual rate of fission is

$$\frac{8}{100} \times 10^7 = 8 \times 10^5 \text{ s}^{-1}$$

Energy released per fission = 200 MeV

Power output of fission $= 8 \times 10^5 \times 200$ MeV s^{-1}

$= 16 \times 10^7$ MeV s^{-1}

Probability of α-particle decay is 92%. Therefore, rate of decay for α-particle is

$$\frac{92}{100} \times 10^7 = 92 \times 10^5 \text{ s}^{-1}$$

For α-decay, the equation is

$$^{248}_{96}Cm \rightarrow ^{244}_{94}Pu + ^{4}_{2}He$$

(248.072220) (244.064100) (4.002603)

Mass of decay products is

$244.064100 + 4.002603 = 248.06673$.

Mass defect,

$\Delta m = 248.072220 - 248.066703 = 0.005517$ u

Energy released per α-decay is

$92 \times 10^5 \times 5.1363$ MeV s^{-1} $= 4.725 \times 10^7$ MeV s^{-1}

Total power output $= 16 \times 10^7 + 4.725 \times 10^7$

$= 20.725 \times 10^7$ MeV s^{-1}

$= 20.725 \times 10^7 \times 1.6 \times 10^{-13}$ J s^{-1}

$= 33.16 \times 10^{-6}$ W $= 33.16\,\mu$W

9. a. The rate of formation of radioactive nuclei is α.

Rate of decay of radioactive nuclei is λN. Therefore,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha - \lambda N} = dt$$

Integrating,

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + A \qquad \text{(i)}$$

where A is constant of integration.

At $t = 0$, $N = N_0$

$$\therefore \quad \frac{\log_e(\alpha - \lambda N_0)}{-\lambda} = A$$

Therefore, Eq. (i) gives

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + \frac{\log_e(\alpha - \lambda N)}{-\lambda}$$

$$\Rightarrow \quad \log_e\left(\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right) = -\lambda t$$

$$\Rightarrow \quad \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t} \qquad \text{(ii)}$$

b. Given, $a = 2N_0\lambda$

$$t = T_{1/2} = \frac{0.693}{\lambda}$$

$$\therefore \quad N = \frac{2N_0\lambda}{\lambda}(1 - e^{-0.693}) + N_0 e^{-0.693}$$

$$= N_0(2 - e^{-0.693}) = N_0(2 - 0.5)$$

$$\therefore \quad N = 1.5\, N_0$$

When $t \to \infty$, Eq. (ii) gives

$$N = \frac{\alpha}{\lambda}$$

10. Total energy produced by the reactor in time $t = 10$ years,

$$E = 1000 \times 10^6 \times 10 \times 3.15 \times 10^7 \text{ J}$$

$$= 3.15 \times 10^{17} \text{ J}$$

$$\text{Efficiency} = \frac{\text{output energy}}{\text{input energy}}$$

$$\Rightarrow \text{Input energy caused by fission} = \frac{\text{output energy}}{\text{efficiency}}$$

$$= \frac{3.15 \times 10^{17}}{(10/100)} = 3.15 \times 10^{18} \text{ J}$$

Energy produced by 1 fission of ^{235}U = 200 MeV

$$= 200 \times 1.6 \times 10^{-13} \text{ J}$$

$$= 3.2 \times 10^{-11} \text{ J}$$

$$\text{Therefore, number of fissions required} = \frac{\text{Total energy}}{\text{Energy per fission}}$$

$$= \frac{3.15 \times 10^{18}}{3.2 \times 10^{-11}}$$

$$\approx 9.8 \times 10^{28}$$

Hence, mass of uranium required is given by

$$m = \frac{N}{N_a} \times 235 \text{ kg} = \frac{9.8 \times 10^{28}}{6.02 \times 10^{26}}$$

$$= 38.2 \times 10^3 \text{ kg}$$

11. de Broglie wavelength of α-particle, $\lambda_\alpha = 5.76 \times 10^{-15}$ m

de Broglie wavelength, $\lambda = \dfrac{h}{p}$

Therefore, momentum of α-particle,

$$p_\alpha = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{5.76 \times 10^{-15}} = 1.15 \times 10^{-19} \text{ kg m s}^{-1}$$

If p_d is momentum of daughter nucleus, then from conservation of linear momentum,

$$p_d = -p_\alpha = -1.15 \times 10^{-19} \text{ kg m s}^{-1}$$

Hence, total kinetic energy of final state,

$$E = E_\alpha + E_d = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_\alpha^2}{2m_\alpha} + \frac{p_d^2}{2m_d}$$

$$= \frac{p_\alpha^2}{2}\left(\frac{1}{m_\alpha} + \frac{1}{m_d}\right) = \frac{p_\alpha^2(m_d + m_\alpha)}{2m_\alpha m_d}$$

$$1 \text{ amu} = 931.470 \text{ MeV}/c^2 = \frac{931.470 \times 1.6 \times 10^{-13}}{(3 \times 10^8)^2} \text{kg}$$

$$= 1.66 \times 10^{-27} \text{ kg}$$

Therefore, total kinetic energy of final state is

$$\frac{(1.15 \times 10^{-19})^2 \times (223.610 + 4.002) \text{ amu}}{2 \times (4.002 \text{ amu}) \times (223.610 \text{ amu})}$$

$$= \frac{(1.15 \times 10^{-19})^2 \times 227.612}{2 \times 4.002 \times 223.610 \times 1.66 \times 10^{-27}} \text{J}$$

$$= \frac{(1.15)^2 \times 227.612}{2 \times 4.002 \times 223.610 \times 1.66} \times 10^{-11} \text{ J}$$

$$= 10^{-12} \text{ J} = \frac{10^{-12}}{1.6 \times 10^{-13}} \text{ MeV} = 6.25 \text{ MeV}$$

Mass of parent nucleus = $m_d + m_\alpha$ − (BE)

$$= \left(223.610 + 4.002 - \frac{6.25}{931.47}\right) \text{amu}$$

$$= (227.612 - 0.007) \text{ amu} = 227.605 \text{ amu}$$

12. a. Let number of nuclei of Y and Z at time $t = t$, are N_Y and N_Z. Then

We can write the rate equations of the populations of X, Y and Z

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \qquad \text{(i)}$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \qquad \text{(ii)}$$

$$\text{and} \quad \left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \qquad \text{(iii)}$$

b. As we are given $N_Y(t) = \dfrac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-l_Y t} - e^{-l_X t}]$

Hence for N_Y to be maximum $\dfrac{dN_Y(t)}{dt} = 0$

i.e., $\lambda_X N_X = \lambda_Y N_Y$ (iv) [from eq. (ii)]

or $$\lambda_X(N_0 e^{-\lambda_X t}) = \lambda_Y \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$$

or $$\frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1 \Rightarrow \frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

or $(\lambda_X - \lambda_Y)t \ln(e) = \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$

or $t = \frac{1}{\lambda_X - \lambda_Y}\ln\left(\frac{\lambda_X}{\lambda_Y}\right)$

Substituting the values of λ_X and λ_{XY}, we have

$$t = \frac{1}{(0.1 - 1/30)}\ln\left(\frac{0.1}{1/30}\right) = 15\ln(3)$$

we get $t = 16.48$ s

c. The population of X at this moment.

$$N_X = N_0 e^{-\lambda_X t} = (10^{20})e^{-(0.1)(16.48)}$$

$$N_X = 1.92 \times 10^{19}$$

$$N_Y = \frac{N_X \lambda_X}{\lambda_Y} \quad \text{[from eq. (iv)]}$$

$$= (1.92 \times 10^{19})\frac{(0.1)}{(1/30)} = 5.76 \times 10^{19}$$

$$N_Z = N_0 - N_X - N_Y$$

$$= 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$$

we get $N_Z = 2.32 \times 10^{19}$

13. Let N_0 be initial number of nuclei at time $t = 0$.
The number of undecayed nuclei in time t is $N = N_0 e^{-\lambda t}$
The number of nuclei decayed in time t is

$$n = N_0 - N = N_0 - N_0 e^{-\lambda t} \quad \text{(i)}$$

The number of undecayed nuclei in next time t.

$$N' = Ne^{-\lambda t} = (N_0 e^{-\lambda t})\, e^{-\lambda t} = N_0 e^{-2\lambda t} \quad \text{(ii)}$$

Number of decayed nuclei in next time t.

$$0.75n = N - N' = N_0 e^{-\lambda t} - N_0 e^{-2\lambda t}$$

$$= N_0 e^{-\lambda t}(1 - e^{-\lambda t}) \quad \text{(iii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$0.75 = e^{-\lambda t} \Rightarrow \frac{4}{3} = e^{\lambda t}$$

Taking natural logarithm,

$$\ln\frac{4}{3} = \lambda t \Rightarrow \lambda = \frac{\ln\frac{4}{3}}{t}$$

Given $t = 2$ s.

$$\therefore \lambda = \frac{\ln 4 - \ln 3}{2} = \frac{2\ln 2 - \ln 3}{2} = \frac{2 \times 0.6931 - 1.0986}{2}$$

$$= 0.1438 \text{ s}^{-1}$$

Mean life, $\tau = \frac{1}{\lambda} = \frac{1}{0.1438} = 7$ s (whole number)

14. ^{238}U atoms decay to form ^{206}Pb atoms. Let N_0 be initial number of U atoms. After time t, let N_U be the number of U atoms left. Then

$$\frac{N_U}{N_0} = \left(\frac{1}{2}\right)^n, \text{ where } n \text{ is number of half-lives}$$

$$n = \frac{t}{T} = \frac{1.5 \times 10^9 \text{ years}}{4.5 \times 10^9 \text{ years}} = \frac{1}{3}$$

$$\therefore N_U = \left(\frac{1}{2}\right)^{1/3} N_0 \quad \text{(i)}$$

Number of Pb atoms, $N_{Pb} = N_0 - N_U$

Hence, required ratio is

$$R = \frac{N_U}{N_{Pb}} = \frac{1}{2^{1/3} - 1} = \frac{1}{1.259 - 1} = \frac{1}{0.259} = 3.86$$

15. Let a = Uranium atom present initially
And $a - x$ = Uranium atoms left after n half life

We have $N = N_0\left(\frac{1}{2}\right)^n$

Here $(a - x) = a\left(\frac{1}{2}\right)^n$

$$n = \frac{t}{t_{1/2}} = \frac{1.5 \times 10^9}{4.5 \times 10^9} = \frac{1}{3}$$

$$a - x = a\left(\frac{1}{2}\right)^{1/3}$$

$$\frac{a}{a - x} = \frac{1}{(1/2)^{1/3}} = \frac{2^{1/3}}{1} = 1.26$$

$$\frac{x}{a - x} = 1.26 - 1 = 0.26$$

16. a. We know that the radius of nucleus is given by the formula $r = r_0 A^{1/3}$ where r_0 = constant. And A = mass number.
For the Nucleus $r_1 = r_0\, 4^{1/3}$
For unknown nucleus $r_1 = r_0\, (4)^{1/3}$

$$\frac{r_2}{r_1} = \left(\frac{A}{4}\right)^{1/3}$$

$$(14)^{1/3} = \left(\frac{A}{4}\right)^{1/3} \Rightarrow A = 56$$

$\therefore$ Number of proton = A − number of neutrons
= 56 − 30 = 26

$\therefore$ Atomic number = 26

b. We know that $v = Rc(z - b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_1^2}\right]$

Here, $R = 1.1 \times 10^7$, $C = 3 \times 10^8$, $Z = 26$ $b = 1$ (for K_α), $n_1 = 1$, $n_2 = 2$

$$v = 1.1 \times 10^7 \times 3 \times 10^8[26 - 1]^2\left[\frac{1}{1} - \frac{1}{4}\right]$$

$$3.3 \times 10^{15} \times 25 \times 25 \times \frac{3}{4} = 1.546 \times 10^{18} \text{ Hz}$$